Computer Networks X_400487

Lecture 3

Chapter 3: The Data Link Layer—Part 2

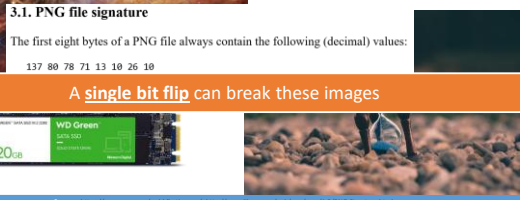
Lecturer: Jesse Donkervliet

Data Link Layer — Roadmap

Part 1
- Framing
- Flow Control
- Guaranteed Delivery
- Sliding Window Protocols

Part 2
- **Error detection**
- **Error correction**

1 Gibibyte = 8×2^{30} bits



A **single bit flip** can break these images

Sources: https://www.nasa.gov/webfiles-images/ ; http://www.libpng.org/pub/png/spec/1.2/PNG-Structure.html ; https://pngcheck.com/photos/PCKVOV18.7z/

Error Detection

Detecting errors in received frames

Q: What causes these bit flips?

Data at sender: 01110101010101110101 0001

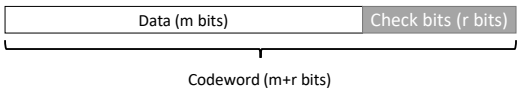
Data at receiver: 01110101**1**1010111010 0001

Somehow bit was flipped!

Adding redundant bits

For a message of m bits,
send an extra r redundant bits.

Send $m + r$ to the receiver. ← Systematic code



Hamming distance

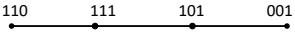
Number of bits that differ between two bit-strings

10001001
10110001

Three bit flips required to change from one sequence to the other.

Adding redundant bits increases the distance between valid bit strings!

Hamming distance: 3

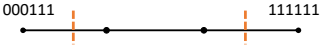


How many errors can we detect?

Consider code words:
000111
111111
000000

Q: How many single-bit errors can we detect?

Hamming distance 3,
so we can detect $3 - 1 = 2$ single-bit errors.



Error detection

Q: How to assess the quality of these codes?

Linear, systematic block codes:
1. Parity
2. Checksums
3. Cyclic Redundancy Checks (CRCs)

Block is $n = m + r$ bits large
Important code properties:
1. Code Ratio: $\frac{m}{n}$
2. How many / which kind of errors are reliably detected?

Parity

Q: How many bit errors can be detected?

Add single bit such that:
- The sum of the data bits modulo 2 is 0
- The number of 1's is even

Send example: 1110000 → 1110000**1**
Receive example: 11010101 ← Error detected!

Easy to detect an *odd* number of errors

Parity

Q: How many bit errors can be detected?

Add single bit such that:
- The sum of the data bits modulo 2 is 1
- The number of 1's is odd

Send example: 1110000 → 1110000**0**
Receive example: 11010100 ← Error detected!

Easy to detect an *odd* number of errors

Multiple parity bits

transmit order →

11100000010101110101001010001111000

Multiple parity bits

transmit order →

1110000 → 1
0010101 → 1
1101010 → 0
0101000 → 0
1111000 → 0

Multiple single-bit errors detected.

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Burst errors

transmit order →

1110000 → 1
0010101 → 1
1101010 → 0
0101000 → 0
1111000 → 0

channel →

Burst error not detected!

0011100 → 1
0010101 → 1
1101010 → 0
0101000 → 0
1111000 → 0

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Burst errors

transmit order →

1110000
0010101
1101010
0101000
1111000
↓↓↓↓↓↓↓
1011111

channel →

Burst error detected.

0011100
0010101
1101010
0101000
1111000
↓↓↓↓↓↓↓
1011111

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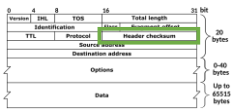
Checksums

Q: Disadvantages?

Checksum treats data as N-bit words and adds N check bits that are the modulo 2^N sum of the words.

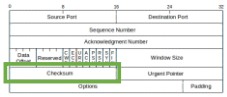
Example: Internet (i.e., IPv4) 16-bit one's complement header checksum



Used in IPv4 (network layer)

Properties:

- Improved error detection over parity bits.
- Detects bursts up to N errors.
- Vulnerable to systematic errors, e.g., added zeros.



Used in TCP (transport layer)

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Checksum Example

Groups of 8 bits → calculate *sum mod 256*

transmit order →

11101000
00100101
11011010
01010000
11111000

add

00000000
11101000
---------- +
11101000

Internet checksum uses one's complement arithmetic

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Checksum Example

Groups of 8 bits → calculate *sum mod 256*

transmit order →

11101000
00100101
11011010
01010000
11111000

add

11101000
00100101
---------- +
100001101

00001110 ← (carry added back: one's complement arithmetic)

Internet checksum uses one's complement arithmetic

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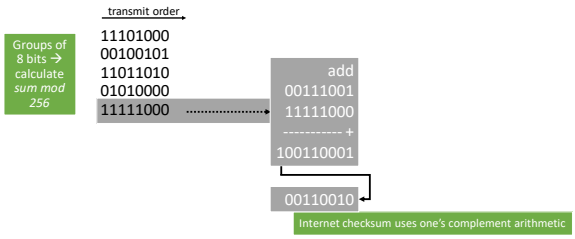
Checksum Example



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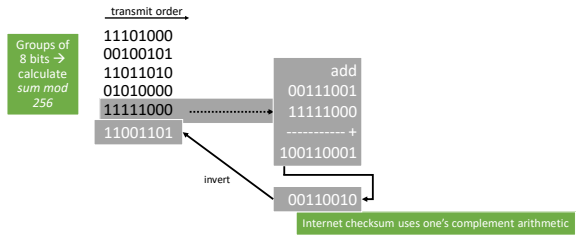
Checksum Example



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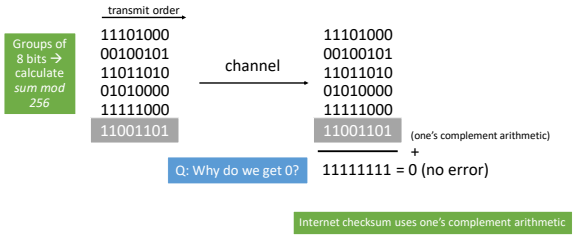
Checksum Example



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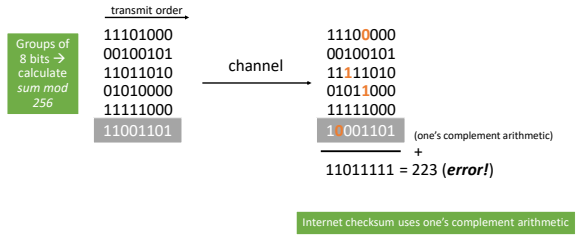
Checksum Example



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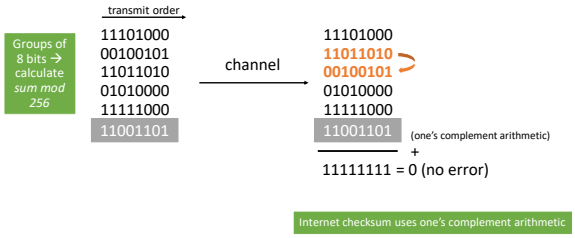
Checksum Example



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Checksum Example #2

To send: AB A8 EF F3

Compute 8-bit (1 byte) word checksum:

```
 AB                          B(11)+8+F(15)+3=37
 A8                          37 = 2×16 + 5
 EF                             = 0x25 (carry = 2)
 F3
 ——+ (1's complement)        2+A(10)+A+E(14)+F=51
??5                          51 = 3×16 + 3
335                             = 0x33
```

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Checksum Example #2

To send: AB A8 EF F3
Compute 8-bit (1 byte) word checksum:

```
AB                              B(11)+8+F(15)+3=37
A8                              37 = 2×16 + 5
EF                                 = 0x25 (carry = 2)
F3
—+ (1's complement)             2+A(10)+A+E(14)+F=51
38  (335 → 3 + 35 = 38)         51 = 3×16 + 3
                                   = 0x33
```


Checksum Example #2

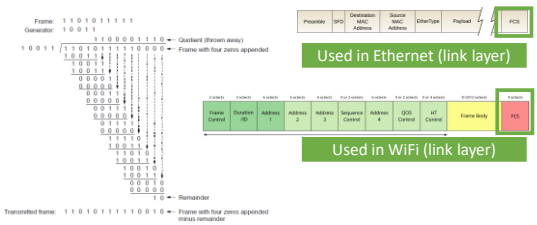
To send: AB A8 EF F3
Compute 8-bit (1 byte) word checksum:

```
AB
A8
EF
F3
—+ (1's complement)
38  (335 → 3 + 35 = 38)
```

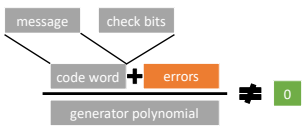
```
Checksum is inverse
of 38:

3 → C (15-3=12=C)
8 → 7 (15-8= 7=7)

Checksum: C7
```


Lecture resumes at 14:18

Cyclic Redundancy Check



Cyclic Redundancy Check
The concept



Cyclic Redundancy Check
Properties and practice

Sender and receiver agree upon polynomial in advance
Example: Ethernet's 33-bit polynomial is:
$x^{32}+x^{26}+x^{23}+x^{22}+x^{16}+x^{12}+x^{11}+x^{10}+x^{8}+x^{7}+x^{5}+x^{4}+x^{2}+x^{1}+1$
CRC is computed with simple shift/XOR circuits
Because we use modulo 2 arithmetic.
Stronger detection than checksums:

1. Can detect all double bit errors, odd bit errors
2. Detect all burst errors $\leq r$ bits (in example, $r = 32$)
3. Not vulnerable to systematic errors
4. …

Cyclic Redundancy Check

Cyclic Redundancy Check Example

Sender adds CRC

$\mathbf{1} \times x^4 + \mathbf{0} \times x^3 + \mathbf{0} \times x^2 + \mathbf{1} \times x^1 + \mathbf{1} \times x^0$

message: 11010101**0000**
generator: 10011

$x^4 + x + 1$

```
10011010000
10011
   10011
   10011
    0011
```

Modulo 2 arithmetic. No carries/borrows

Q: Consequences for implementation?

Message: 11010101, CRC: 0011,
Codeword: 11010101**0011**

$\frac{11010101 0011}{10011} = 0$

Cyclic Redundancy Check Example

Receiver checks for errors

message: 11010101**0011**
generator: 10011

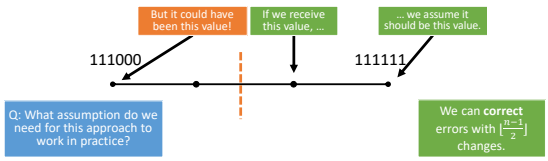
```
10011010011
10011
   10011
   10011
       0
```

$\frac{11010101 0011}{10011} = 0$, no error

Error Correction

How many errors can we *correct*?

Consider a code with hamming distance n.
We have seen that we can **detect** $n - 1$ single-bit errors.



Error correction

1. Hamming codes
2. Binary convolutional codes
3. Reed-Solomon codes
4. Low-Density Parity Check codes
5. … (many others)

Multiple parity bits

receive order →

1110000 → 1
0010101 → **0**
1101010 → 0
0101000 → 0
1111000 → 0

Error in second row!

Multiple parity bits

receive order →

1110000
0010101
1101010
0101000
1111000
↓↓↓↓↓↓↓
1010111

Error in fourth column!

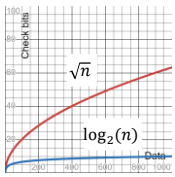
Multiple parity bits

receive order →

1110000 → 1
0010101 → **0**
1101010 → 0
0101000 → 0
1111000 → 0
↓↓↓↓↓↓↓
1010111

Error located!

= Row+column parity bits
= Hamming code



Hamming codes

Minimum number of check bits such that *all* 1-bit errors can be *corrected*!

Example of an (11, 7) Hamming code correcting a single-bit error.

Hamming codes
An example

Use bit-locations that are a power of 2 as check bits. Use the remaining positions for the message.

message: 11010101
codeword: __1_101_0101
positions: 123456789…

Hamming codes
An example

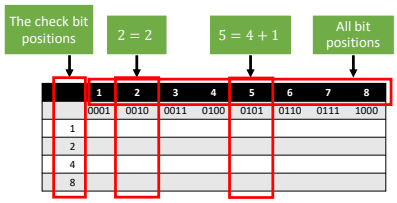
Use bit-locations that are a power of 2 as check bits. Use the remaining positions for the message.

message: 1 101 0101
codeword: __1_101_0101
positions: 123456789…

1. Expand all bit locations into powers of two.
2. Decide the value of each check bit in position 2^i by calculating the parity function over all bits that have 2^i in their expansion.

Hamming codes
An example

1. Expand all bit locations into powers of two.



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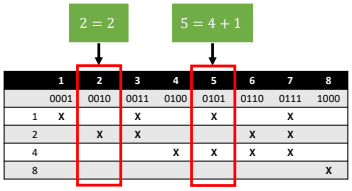
Hamming codes
An example

1. Expand all bit locations into powers of two.



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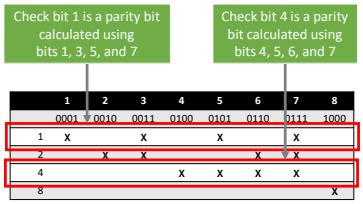
Hamming codes
An example

2. Calculate the parity bit using all bits that have 2^i in their expansion



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Hamming codes
An example

2. Calculate the parity bit using all bits that have 2^i in their expansion

Use bit-locations that are a power of 2 as check bits.
Use the remaining positions for the message.

message: 1 101 0101
codeword: __1_101_0101
positions: 123456789…

Position 1+2
Position 4
Position 4+2+1

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Hamming codes
An example

Use bit-locations that are a power of 2 as check bits. Use the remaining positions for the message.

message: 1 101 0101
codeword: __1_101_0101
positions: 123456789…

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Hamming codes
An example

Use bit-locations that are a power of 2 as check bits. Use the remaining positions for the message.

message: 1 101 0101
codeword: 1_1_101_0101
positions: 123456789…

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Hamming codes
An example

Use bit-locations that are a power of 2 as check bits. Use the remaining positions for the message.

message: 1 101 0101

codeword: 1_**1**_101_**0**1**0**1

positions: 1**2**3456789…

Hamming codes
An example

Use bit-locations that are a power of 2 as check bits. Use the remaining positions for the message.

message: 1 101 0101

codeword: 1**1**1_101_**0**1**0**1

positions: 1**2**3456789…

Hamming codes
An example

Use bit-locations that are a power of 2 as check bits. Use the remaining positions for the message.

message: 1 101 0101

codeword: 111110100101

positions: 123456789…

Hamming codes
Error correction

Use bit-locations that are a power of 2 as check bits.
Use the remaining positions for the message.

message: 1 101 0101 ← Single-bit error

codeword: 111111100101

positions: 123456789…

Computer *error syndrome*:

Check bit 1 = parity of bits 1, 3, 5, 7, 9, 11

Hamming codes
Error correction

Use bit-locations that are a power of 2 as check bits.
Use the remaining positions for the message.

message: 1 101 0101 ← Single-bit error

codeword: 1_1_1_1**1**00_101

positions: 123456789…

Computer *error syndrome*:

Check bit 1 = parity of bits 1, 3, 5, 7, 9, 11 = 0

Hamming codes
Error correction

Use bit-locations that are a power of 2 as check bits.
Use the remaining positions for the message.

message: 1 101 0101 ← Single-bit error

codeword: 1_1_1_1**1**00_101

positions: 123456789…

Computer *error syndrome*:

Check bit 1 = parity of bits 1, 3, 5, 7, 9, 11 = 0

Check bit 2 = parity of bits 2, 3, 6, 7, 10, 11 = 1

Hamming codes
Error correction

Use bit-locations that are a power of 2 as check bits.
Use the remaining positions for the message.

message: 1 101 0101
codeword: 111111100101
positions: 123456789…

Single-bit error
Error at location: 110 (binary)

Computer *error syndrome*:

Check bit 1 = parity of bits 1, 3, 5, 7, 9, 11 = 0
Check bit 2 = parity of bits 2, 3, 6, 7, 10, 11 = 1
Check bit 4 = parity of bits 4, 5, 6, 7, 12 = 1

Hamming codes
Error correction

Use bit-locations that are a power of 2 as check bits.
Use the remaining positions for the message.

message: 1 101 0101
codeword: 111111100101
positions: 123456789…

Single-bit error
Error at location: 110 (binary)

Computer *error syndrome*:

Check bit 1 = parity of bits 1, 3, 5, 7, 9, 11 = 0
Check bit 2 = parity of bits 2, 3, 6, 7, 10, 11 = 1
Check bit 4 = parity of bits 4, 5, 6, 7, 12 = 1

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Lecture 3
Chapter 3: The Data Link Layer—Part 2

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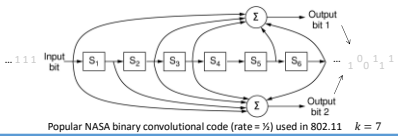
Extra Slides

Convolutional codes

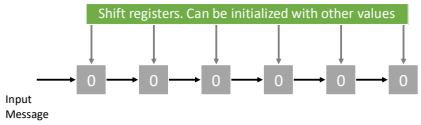
Different from *systematic* codes and *block* codes

Operates on a stream of bits, keeping internal state.
Output stream is a function of last k preceding input bits.
Bits are decoded with the Viterbi algorithm.

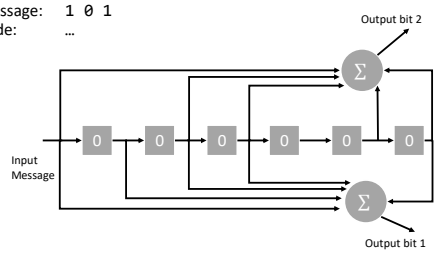
Determines most likely input for given output.



Popular NASA binary convolutional code (rate = ½) used in 802.11 $k = 7$



Message: 1 0 1
Code: …



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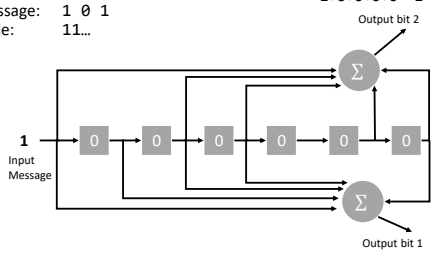
Message: 1 0 1
Code: 11…

1+0+0+0+0 = 1

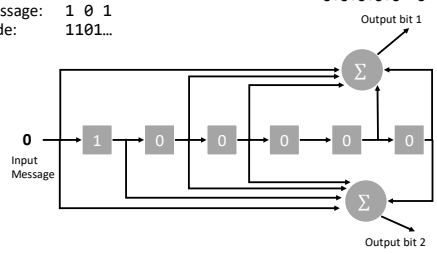


1+0+0+0+0 = 1

64

Message: 1 0 1
Code: 1101…

0+0+0+0+0 = 0



0+1+0+0+0 = 1

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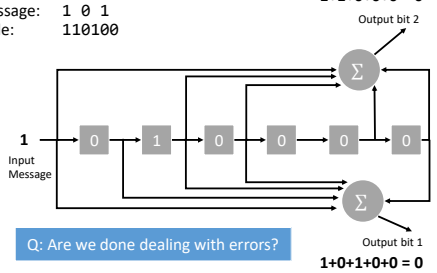
Message: 1 0 1
Code: 110100

1+1+0+0+0 = 0



Q: Are we done dealing with errors?

1+0+1+0+0 = 0

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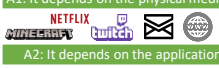
Data Link Layer Summary

A1: It depends on the physical medium



A2: It depends on the application

Framing (byte stuffing, bit stuffing, etc)
Guaranteed delivery Q: When needed?
• Sequence numbers, acknowledgments, and retransmissions
Flow control Q: When needed?
• Stop-and-Wait
• Sliding Window
Error control Q: When needed?
• Detection (e.g., Parity bit, Checksum, CRC, …)
• Correction (e.g., Hamming Code, Convolutional Code, …)

A3: We may want to address the problem in a higher layer
Q: Can you think of an example?

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